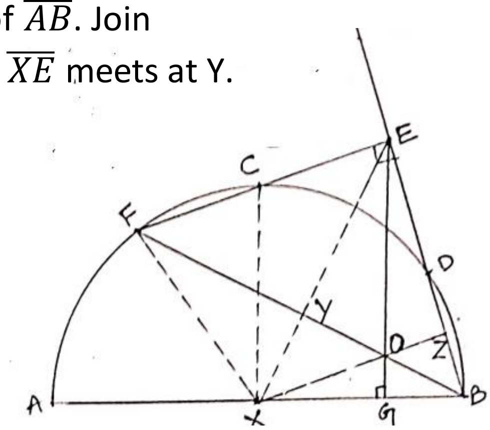
# II Prize Winner – Mr.Hrudananda Bhoi's Solution

**Given:**

Arc ACB is a semicircle where C is the midpoint of the arc ACB. D is any point on the arc BC and CE is perpendicular to extended $\overline{BD}$ at E. $\overline{EG} \perp \overline{AB}$ and extended $\overline{EC}$ meets the arc AC at F. $\overline{FB}$ and $\overline{EG}$ meets at O. We have to prove that EO=$\frac{1}{2}AB$.

**Construction :**

Let X be the centre of the semicircle ie the midpoint of $\overline{AB}$. Join $\overline{FX}, \overline{EX}, \overline{CX}, \overline{OX}$. Extended $\overline{OX}$ meets $\overline{BE}$at Z. $\overline{FB}$ and $\overline{XE}$ meets at Y.




**Proof:**

C is the midpoint of arc ACB and X is the centre.

$\Rightarrow \overline{CX} \perp \overline{AB}$, *But* $\overline{EG} \perp \overline{AB}$ *(Given)*

$\Rightarrow \overline{CX} \parallel \overline{EG}$

Now, $m\angle BXC = 90° \Rightarrow m\angle BFC = \frac{1}{2} m\angle BXC = 45°$

In the right angled $\Delta BEF$ $m\angle BFC = 90°,\ m\angle BFE = 45°$

$\Rightarrow\ m\angle EBF = 45° \ \Rightarrow BE = EF$

Now, BE=EF and FX=BX (radius)

$\Rightarrow$ BXFE is a kite and hence $\overline{XE} \perp \overline{BF}$

$\Rightarrow \overline{BY} \perp \overline{XE}$, Further $\overline{EG} \perp \overline{BX}$

$\Rightarrow O$ is the orthocentre of $\Delta BEX$

$\Rightarrow \overline{XOZ} \perp \overline{BE} \implies \overline{XZ} \parallel \overline{CE}$ (As $\overline{CE} \perp \overline{BE}$ & $\overline{XZ} \perp \overline{BE}$ )

$\Rightarrow$ CEOX is a parallelogram (As $\overline{CE} \parallel \overline{XO}$ & $\overline{CE} \parallel \overline{EO}$)

$\Rightarrow CX = EO$, But CX = $\frac{1}{2} AB$ (CX radius & AB diameter)

$\Rightarrow$EO = $\frac{1}{2}$ ***AB*** ______________________________ **Proved**